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THE EFFECT OF A MAGNETIC FIELD ON FREE CONVECTION HEAT TRANSFER

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Abstract—Free convection heat transfer due to the simultaneous action of buoyancy and induced magnetic forces is investigated. The analysis is carried out for laminar boundary-layer flow about an isothermal vertical plate. It is found that the free convection heat transfer to liquid metals may be significantly affected by the presence of a magnetic field; but that very small effects are experienced by other fluids.

Résumé—Cet article étudie la transmission de chaleur par convection libre résultant de l'action simultanée des forces de convection naturelle et de forces magnétiques induites. L'étude a été faite pour un écoulement de couche limite laminaire sur une plaque verticale isotherme. On a trouvé que, pour des métaux liquides la transmission de chaleur par convection libre pouvait être notablement affectée par la présence d'un champ magnétique; avec d'autres fluides ces effets sont très petits.

Zusammenfassung—Es wurde der Wärmeübergang durch freie Konvektion bei gleichzeitiger Einwirkung von Auftriebs- und induzierten magnetischen Kräften untersucht, und zwar für die laminare Grenzschicht an der senkrechten isothermen Platte. Es zeigt sich ein starker Einfluss des Magnetfeldes auf den Wärmeübergang bei flüssigen Metallen, bei anderen Flüssigkeiten dagegen nur ein sehr geringer.

Аннотация—В статье рассматривается иеренос тепла под действием свободной конвекции и индуцированного магнитного поля. Исслэдуется ламинарный пограничный слой вокруг изотермической вертикальной пластины. Установлено, что на перенос тепла при свободной конвекции в жидких металлах существенно влияет наличие магнитного поля, в то время как на другие жидкости влияние магнитного поля небольшое.

NOMENCLATURE

- $\mathbf{B}$, magnetic induction vector;
- B_0, externally imposed y-component of $\mathbf{B}$;
- $\mathbf{F}$, induced magnetic force;
- F_x, x-component of $\mathbf{F}$;
- f, transformed stream function equation (10);
- f_0, f_1, functions of η;
- Gr, Grashof number, $g\beta|T_w - T_\infty|x^3/\nu^2$;
- g, acceleration due to gravity;
- $\mathbf{J}$, current density vector;
- k, thermal conductivity;
- Pr, Prandtl number, ν/α;
- Q, overall rate of heat transfer;
- Q_0, Q in absence of magnetic field;
- q, local rate of heat transfer per unit area;
- q_0, q in absence of magnetic field;
- T, static temperature; T_∞, ambient temperature; T_w, wall temperature; $\Delta T = T_w - T_\infty$;
- u, velocity component in x direction;
- $\mathbf{V}$, velocity vector;
- v, velocity component in y direction;
- x, co-ordinate measuring distance from leading edge;
- y, co-ordinate measuring distance normal to plate.

Greek symbols

- α, thermal diffusivity;

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β, thermal expansion coefficient, $-1/\rho\,(\partial\rho/\partial T)_p$;
η, similarity variable, equation (9a);
θ, dimensionless temperature, $(T - T_\infty)/(T_w - T_\infty)$;
θ_1, θ_2, functions of η;
ν, kinematic viscosity;
ξ, magnetic parameter, equation (9b);
ρ, density;
σ, electrical conductivity;
ψ, stream function.

INTRODUCTION

It is common to classify buoyancy forces and induced magnetic forces among the body forces which may occur in fluid mechanics problems. The separate action of each of these forces in establishing and modifying fluid flows has been studied rather extensively. In the present investigation, consideration is given to the situation where buoyancy and magnetic forces act simultaneously.

The specific problem selected for study is the flow and heat transfer in an electrically-conducting fluid adjacent to an isothermal vertical plate. The configuration is pictured schematically in Fig. 1. The plate surface is maintained at a uniform temperature T_w which may either exceed the ambient temperature T_∞, Fig. 1(a), or may be less than T_∞, Fig. 1(b). When $T_w > T_\infty$, an upward flow is established along the plate due to free convection; while when $T_w < T_\infty$, there is a downflow. Additionally, a magnetic field B_0 acts normal to the plate surface. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force which tends to oppose the fluid motion. Consequently, the resultant flow

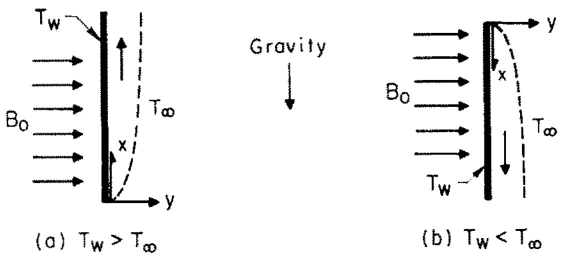

Fig. 1. Physical model and co-ordinates.

and the corresponding heat transfer should be decreased relative to the case of pure free convection.

There is an interesting distinction between the present problem and others involving magnetohydrodynamic effects in *forced convection* boundary layers. In the latter situation, induced magnetic forces may modify the free stream flow and in turn, this may effect the external pressure gradient or the free stream velocity which is imposed on the boundary layer. Thus, a complete boundary-layer solution would involve a magnetohydrodynamic solution for the inviscid free stream (for example, see Refs. 1 and 2). In some cases, the interaction between the free stream flow and the magnetic field has been treated in an incomplete manner, giving rise to an ambiguity in the boundary-layer results [3]. On the other hand, in the free convection problem, the velocity is zero in the ambient fluid and induced magnetic forces do not exist there. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. As a consequence, the free convection problem may be formulated in a simpler and perhaps more exact manner than the forced convection problem.

The analysis is carried out for the case of uniform surface temperature T_w and uniform imposed magnetic field B_0 (independent of x). These conditions do not lead to a similar solution of the laminar boundary-layer equations. Therefore, solutions of the governing equations have been obtained utilizing a series expansion method. At the time this study was carried out, there were no analyses of the magnetohydrodynamic free convection problem in the readily-available literature. During the process of review by the Journal, some additional references were brought to the attention of the authors. The magnetohydrodynamic free convection problems considered therein are complementary but different from the situation considered here. In Ref. 4 the isothermal vertical plate is analyzed under the assumption that the imposed magnetic field B_0 varies as $x^{-1/4}$. This is the condition required to achieve a similarity boundary layer.

Approximate solutions were obtained by the Kármán–Pohlhausen integral method. Refs. 5 and 6 also consider similarity-type solutions. In Ref. 5, the isothermal wall case is solved numerically for various Prandtl numbers and magnetic parameters and additionally, asymptotic solutions are given. Ref. 6 explores the relationship between the x-dependence of wall temperature and imposed magnetic field which provides similarity conditions, but numerical results are not given. An imposed field B_0 varying as $x^{1/4}$ along an isothermal plate was considered in Ref. 7. This did not lead to a similarity-type boundary layer and a series expansion method was utilized. The results of Ref. 7 are somewhat in doubt inasmuch as an error was discovered in one of the governing differential equations [5].

ANALYSIS

Mathematical formulation

The starting point of the analysis is the basic conservation laws of mass, momentum, and energy. To obtain the mathematical statement of these laws, we utilize the well-known governing equations for free convection (for example, p. 327 of Ref. 8) to which are added terms appropriate to the magnetic effects. Characterizing the induced magnetic force by **F**, we write

mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \pm\, g\beta\,(T - T_\infty) + \frac{F_x}{\rho} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

energy:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}\,. \tag{3}$$

The plus-minus sign has been attached to the buoyancy force $g\beta(T - T_\infty)$ so that the equations are valid for both the upflow and downflow situations pictured on Fig. 1(a) and 1(b) respectively. Fluid property variations have been considered only to the extent of a density variation which provides a buoyancy force. Viscous dissipation has been neglected in the energy equation (3). Further, since the Joule heating (electrical dissipation) is usually the same order as the viscous dissipation, it too has been neglected.

The magnetic force **F** may, in the absence of excess charges, be written as [9]

$$\mathbf{F} = \mathbf{J} \times \mathbf{B} \tag{4}$$

where **J** and **B** are respectively the current density and magnetic induction vectors. Further, when the external electric field is zero and the induced electric field negligible, the current density is related to the velocity by Ohm's Law as follows

$$\mathbf{J} = \sigma(\mathbf{V} \times \mathbf{B}) \tag{5}$$

where σ denotes the electrical conductivity of the fluid. Next, under the condition that the magnetic Reynolds number is small, the induced magnetic field is negligible compared with the applied field. This condition is usually well satisfied in terrestrial applications, especially so in (low-velocity) free convection flows. So, we write

$$\mathbf{B} = \mathbf{i}_z\, B_0. \tag{6}$$

Bringing together equations (4), (5), and (6), the force component F_x is found to be

$$F_x = -\sigma u B_0^2 \tag{7}$$

and this may then be introduced into the momentum equation (2).

The specification of the boundary conditions is necessary to complete the statement of the problem. At the surface, the velocities are zero to satisfy the conditions of no slip and an impermeable wall. In addition, temperature continuity requires that fluid and solid share the same temperature. Far from the surface, the velocity approaches zero and the temperature approaches that of the surroundings. Formally, these conditions may be stated as

$$\left.\begin{array}{l} u = v = 0 \\ T = T_w = \text{const.} \end{array}\right\} y = 0 \qquad \left.\begin{array}{l} u \to 0 \\ T \to T_\infty \end{array}\right\} y \to \infty\,. \tag{8}$$

Having thus completed the statement of the problem, attention may next be directed toward finding a solution. The first thought would be to seek a similarity solution; that is, to seek a form of solution in which the velocity and temperature profiles maintain a similar shape at all values of position x. Mathematically speaking, the desirable property of such a solution is that the governing equations (1), (2), (3), and (7) become ordinary differential equations. However, it is found that the conditions that B_0 and T_w are both independent of x cannot be satisfied by a similarity solution. As a consequence, some other approach must be found.

Series solution

Previous experience with non-similar boundary layer problems suggests that a series solution might be useful in the present situation. To this end, the following new co-ordinates are introduced

$$\eta = \frac{y}{x^{1/4}}\left[\frac{g\beta|T_w - T_\infty|}{4\nu^2}\right]^{1/4} = \frac{y}{x}\left(\frac{Gr}{4}\right)^{1/4} \tag{9a}$$

$$\xi = \frac{2\sigma B_0^2}{\rho[g\beta|T_w - T_\infty|]^{1/2}}\, x^{1/2} = \frac{\sigma B_0^2 x^2/\rho\nu}{(Gr/4)^{1/2}} \tag{9b}$$

where Gr is the Grashof number. The variable η is immediately recognized as the free convection similarity variable. On the other hand, ξ, which is essentially a stretched x co-ordinate, is an index to the relative importance of the magnetic forces. Next, new dependent variables f and θ are defined as

$$f(\xi, \eta) = \frac{\psi}{[64\, g\beta\nu^2|T_w - T_\infty|\,]^{1/4}\, x^{3/4}},$$

$$\theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

where ψ, the stream function, is related to the velocities u and v in the usual manner

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x}. \tag{11}$$

This definition of ψ immediately satisfies the conservation of mass equation.

We may then proceed to transform the conservation of momentum and energy equations into the new co-ordinates. To facilitate the transformation, it is useful to have the velocity components explicitly expressed in terms of the new variables. From equation (11) in conjunction with (10) and (9), there is obtained

$$u = 2[g\beta|T_w - T_\infty|]^{1/2}\, x^{1/2}\frac{\partial f}{\partial\eta} \tag{12a}$$

$$v = -\frac{[64\, g\beta\nu^2|T_w - T_\infty|\,]^{1/4}}{x^{1/4}}\left[\frac{3}{4}f - \frac{1}{4}\eta\frac{\partial f}{\partial\eta} + \frac{1}{2}\xi\frac{\partial f}{\partial\xi}\right]. \tag{12b}$$

With these, and utilizing the definitions of η, ξ, and θ from equations (9) and (11), the conservation equations (2) and (3) becomes

momentum:

$$2\left(\frac{\partial f}{\partial\eta}\right)^2 - 3f\frac{\partial^2 f}{\partial\eta^2} + 2\xi\left(\frac{\partial^2 f}{\partial\xi\partial\eta}\frac{\partial f}{\partial\eta} - \frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2} + \frac{1}{2}\frac{\partial f}{\partial\eta}\right) = \theta + \frac{\partial^3 f}{\partial\eta^3} \tag{12}$$

energy:

$$-3f\frac{\partial\theta}{\partial\eta} + 2\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{\partial\theta}{\partial\eta}\frac{\partial f}{\partial\xi}\right) = \frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2}. \tag{13}$$

It may be noticed that in the absence of a magnetic field, i.e. $\xi = 0$, these equations reduce to the standard free convection equations as first derived by Schmidt and Beckmann (p. 447 of Ref. 10).

Notwithstanding their change in form, equations (12) and (13) are, like their predecessors (2) and (3), partial differential equations. In order to reduce the problem to one involving more tractable ordinary differential equations, we expand the variables f and θ in series as follows

$$f(\xi, \eta) = f_0(\eta) + \xi f_1(\eta) + \xi^2 f_2(\eta) + \dots \tag{14}$$

$$\theta(\xi, \eta) = \theta_0(\eta) + \xi\theta_1(\eta) + \xi^2\theta_2(\eta) + \dots \tag{15}$$

These may be substituted into equations (12) and (13) and terms grouped according to powers of ξ. From this, there is obtained the governing equations for the f_0, θ_0, f_1, θ_1, ... as

$$\left.\begin{aligned} &f_0''' + 3f_0f_0'' - 2(f_0')^2 = -\theta_0 \\ &\theta_0'' + 3Pr\, f_0\theta_0' = 0 \end{aligned}\right\} \tag{16}$$

$$\left.\begin{aligned} f_1''' + 3f_0f_1'' - 6f_0'f_1' + 5f_0''f_1 = f_0' - \theta_1 \\ \frac{1}{Pr}\theta_1' + 3f_0\theta_1' - 2f_0'\theta_1 = -5f_1\theta_0' \end{aligned}\right\} \quad (17)$$

where primes represent differentiation with respect to η and Pr denotes the Prandtl number.

For computational purposes, the series were truncated after the θ_1 and f_1 terms. Formulated in this way, the problem takes the form of a basic free convection flow upon which the effects of the magnetic field constitute a disturbance. From the numerical results to be shown later, it would appear that truncation after the second term is not a too serious restriction.

The boundary conditions (8) may also be rephrased in terms of the new variables. Utilizing the velocity expressions (12) and the definition of θ, equation (10), it is found that

$$\left.\begin{aligned} f_0(0) = f_0'(0) = 0, \quad \theta_0(0) = 1 \\ f_0' \to 0, \quad \theta_0 \to 0 \quad \text{as } \eta \to \infty \end{aligned}\right\} \quad (16a)$$

$$\left.\begin{aligned} f_1(0) = f_1'(0) = \theta_1(0) = 0 \\ f_1' \to 0, \quad \theta_1 \to 0 \quad \text{as } \eta \to \infty \end{aligned}\right\}. \quad (17a)$$

It may be seen that equation (16) and the corresponding boundary conditions (16a) coincide with the final ordinary differential equations for pure free convection. Numerical solutions of these equations covering the Prandtl number range 0·003 to 1000 have been tabulated in detail in Refs. 11 and 12. The computational task thus remaining for the present study was to solve equations (17) subject to the boundary conditions (17a).

By inspection of these equations, it is seen that since f_1 and θ_1 appear in both equations, simultaneous solution is required. Additionally, it is necessary to utilize the solutions of equations (16) as input data, since f_0, f_0', f_0'', and θ_0', all appear in equations (17). Analytical solutions of equations (17) could not be found, and it was necessary to use numerical means. Solutions were carried out on an IBM 704 electronic digital computer for Prandtl numbers of 0·02, 0·72, and 10. Input data for the lowest of these was taken from Ref. 12, while that for the higher Prandtl numbers came from Ref. 11. From the solutions, the essential information which is needed in the heat transfer computation is $(d\theta_1/d\eta)_{\eta=0}$. These results are listed in Table 1, along with the values of $(d\theta_0/d\eta)_{\eta=0}$ as taken from the references.

Table 1. Temperature derivatives

Pr	$\theta_0'(0)$	$\theta_1'(0)$	$\theta_1'(0)/\theta_0'(0)$
10	−1·169	0·125	−0·107
0·72	−0·5046	0·0618	−0·123
0·02	−0·1116	0·0152	−0·137

There are two aspects of these results which deserve mention. First is the fact that $\theta_1'(0)/\theta_0'(0)$ varies only slightly with Prandtl number over the very wide range considered here. This suggests that one may interpolate between or extrapolate beyond the tabulated results without fear of incurring large errors. The second relates to the small magnitude of $\theta_1'(0)/\theta_0'(0) \sim 0·1$. Hence, only with fairly large ξ values (e.g. ~2) will the second term of the series begin to be important.

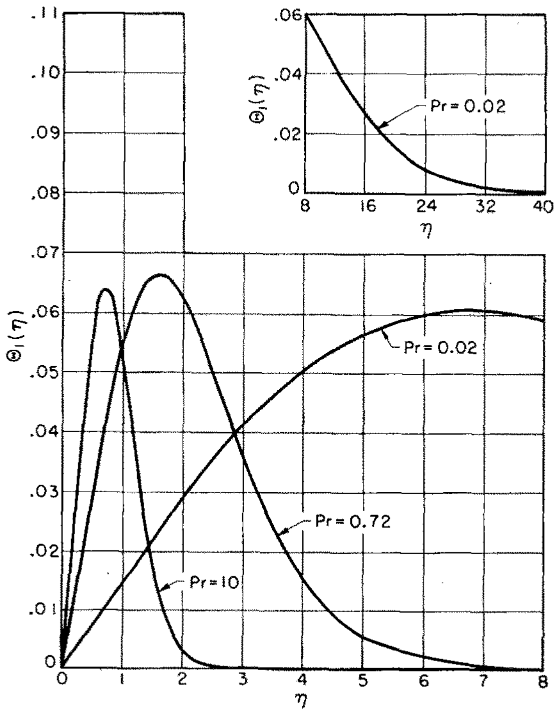

FIG. 2. Variation of the function $\theta_1(\eta)$.

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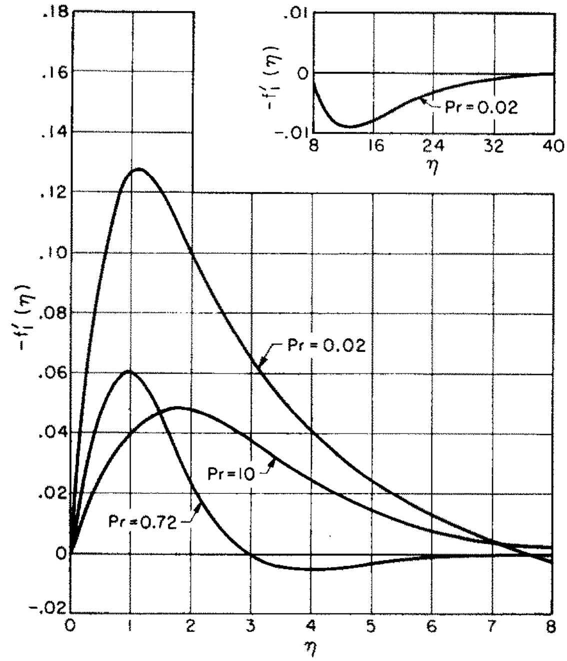

FIG. 3. Variation of the function $f_1'(\eta)$.

Thus, even with truncation after the second term, the theory appears to be able to accommodate fairly large ξ. Of course, small ξ values are accommodated without question.

In addition to the tabulated temperature derivatives which will be used in the heat transfer computation, it appears worthwhile for the sake of completeness to give more detailed information on the new solutions which have been obtained. To this end, the functions $\theta_1(\eta)$ and $f_1'(\eta)$ have been respectively plotted on Figs. 2 and 3 for Prandtl numbers of 0·02, 0·72 and 10. The θ_1 function is associated with the temperature distribution by equation (15); while the f_1' function is associated with the distribution of the velocity component u by equations (12a) and (14). The functions θ_0 and f_0' which are additionally needed in the determination of the temperature and velocity profiles are given in Refs. 11 and 12.

HEAT TRANSFER RESULTS

The local rate of heat transfer flowing from the surface to the fluid may be calculated by Fourier's Law

$$q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{18}$$

Utilizing the definitions of θ and η from equations (10) and (9a), the expression for q can be rephrased as

$$q = -\frac{k}{x}(T_w - T_\infty)\left(\frac{Gr}{4}\right)^{1/4}\left[\frac{d\theta}{d\eta}\right]_{\eta=0}. \tag{19}$$

Then, introducing the series expansion (15) for θ, there is obtained

$$q = \frac{k}{x}(T_w - T_\infty)\left(\frac{Gr}{4}\right)^{1/4} [-\theta_0'(0) - \xi\theta_1'(0) - \ldots] \tag{20}$$

In the absence of a magnetic field, the local heat transfer rate q_0 is given by

$$q_0 = \frac{k}{x}(T_w - T_\infty)\left(\frac{Gr}{4}\right)^{1/4}[-\theta_0'(0)]. \tag{21}$$

The effect of the magnetic field upon the heat flux is then obtained by combining equations (20) and (21), with the result

$$\frac{q}{q_0} = 1 + \left[\frac{\theta_1'(0)}{\theta_0'(0)}\right]\frac{2\sigma B_0^2}{\rho[g\beta|T_w - T_\infty|]}x^{1/2} + \ldots \tag{22}$$

where ξ has been replaced according to equation (9b).

In appraising the influence of the magnetic field, reference is made to the information listed in Table 1. Since $\theta_1'(0)/\theta_0'(0)$ is negative, it follows that the magnetic field reduces the heat transfer, as was expected on physical grounds. Additionally, it is seen that since $\theta_1'(0)/\theta_0'(0)$ is of the order of 0·1, the presence of the magnetic field is significant only if

$$\frac{2\sigma B_0^2 x^{1/2}}{\rho[g\beta|T_w - T_\infty|]^{1/2}}$$

is of the order of one or two.

For 8 N sulphuric acid (1·9 mhos/in) under the reasonable conditions $\Delta T = 50°F$ and $x = 1$ ft, a magnetic field of 25 000 gauss would be required to have a significant effect on heat transfer. Such a magnetic field is exceedingly large and may be considered outside the range of ordinary laboratory practice. For salt water

(0·64 mhos/in), similar field strengths are required to influence the heat transfer, and even larger fields would be needed for ordinary tap water. In the case of gases, electrical conductivities of technically-interesting magnitudes are not achieved until the gas temperatures are very high. For example, even at a temperature of 5500°F and a density corresponding to 100 000 ft altitude (1/70 of standard sea level density), the electrical conductivity of air is only about 10^{-4} mhos/in (Ref. 13, Fig. D, 34a). For these conditions, a magnetic field of perhaps 20 000 gauss is needed to significantly effect the free convection heat transfer. The combination of such high temperatures and high magnetic fields is difficult to achieve and is not commonly encountered. Somewhat lower temperatures and smaller magnetic fields would be required if the air were seeded with potassium.

At the other end of the scale from these illustrations is the case of liquid metals. For liquid mercury ($2·5 \times 10^4$ mhos/in), a 25 per cent reduction in the local heat flux can be achieved with a magnetic field of 1000 gauss for $\Delta T = 50$°F and $x = 3$ in. Thus, it would seem that among all the fluids, the liquid metals appear to be most susceptible to the effects of a magnetic field.

Thus far, consideration has been given to the local heat flux q. The overall rate of heat transfer Q from a section of plate from $x = 0$ to $x = x$ may be calculated by integrating as follows

$$Q = \int_0^x q \mathrm{d}x. \tag{23}$$

Substituting for q from equation (20) and noting that $Gr^{1/4} \sim x^{3/4}$ and $\xi \sim x^{1/2}$, there is obtained

$$\frac{Q}{Q_0} = 1 + \frac{3}{5}\left[\frac{\theta_1'(0)}{\theta_0'(0)}\right]\frac{2\sigma B_0^2}{\rho[g\beta|T_w - T_\infty|]^{1/2}}\,x^{1/2} + \ldots \tag{24}$$

where Q_0, the overall heat flux for pure free convection, is given by

$$Q_0 = \tfrac{4}{3}\,k(T_w - T_\infty)\left(\frac{Gr}{4}\right)^{1/4}[-\theta_0'(0)]. \tag{25}$$

By comparing equations (22) and (24), it is seen that the presence of the magnetic field has a lesser effect on the overall heat transfer than on the local heat transfer.

Equations (22) and (24) may be regarded as computational formulas for q and Q provided that q_0 and Q_0 are known. These may be evaluated from equations (21) and (25) in conjunction with the $\theta_0'(0)$ values of Table 1. When equations (22) and (24) are used for Prandtl numbers other than those considered here, then results for q_0 and Q_0 are available in Refs. 11 and 12, the former being most useful in the range of high Prandtl numbers and the latter for the liquid metal range.

Comparison with other solutions

It is interesting to see how the results from the present analysis relate to those of other investigations. As previously mentioned, the only reliable results currently available are those for the isothermal plate with a magnetic field which varies as $x^{-1/4}$ (i.e. the similarity case). Although this is different from the uniform field case considered here, it still may be interesting to compare the heat transfer results. One of the possible comparisons is to look at the local heat transfer predictions under the condition that the local values of the magnetic parameter ξ in the two analyses (z in Ref. 5) are identical. The local equality of the ξ parameters at a given x can be achieved by arranging the magnetic field strengths to have identical values at that x.* Making use of the analogue computer solutions of Ref. 5 for the similarity case and of equation (22) and Table 1 for the uniform field case, remarkably close agreement between the heat transfer predictions was found for all Prandtl numbers for ξ up to and including unity. This finding suggests that, at least for situations where the magnetic field has a moderate effect on heat transfer (~10–15 per cent), the local heat transfer reduction is not much influenced by the details of the upstream variation of magnetic field. For ξ values exceeding unity, the analogue computer solutions for the similarity case are available only for a Prandtl number of 0·73. For these higher ξ values, the agreement between the local heat

* The field strength for the similarity case would therefore be larger at smaller values of x.

transfer results from the two analyses is not so good, those from the present study being lower. This deviation may be due either to the differences in the upstream magnetic field variation or perhaps, to the series truncation of equation (22). For the more important low Prandtl range, only approximate results are available for the similarity case for $\xi > 1$.

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